NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1348

CRITICAL SHEAR STRESS OF CURVED RECTANGULAR PANELS

By S. B. Batdorf, Manuel Stein, and Murry Schildcrout

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Langley Field, Va.

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SUMMARY

A solution based upon small-deflection theory is presented for the critical shear stress of curved rectangular panels with simply supported edges. Computed curves which cover a wide range of panel dimensions are presented; these curves are found to be in good agreement with test results. Estimated curves are also given for panels with clamped edges.

INTRODUCTION

A series of papers has been prepared to provide information on the buckling of curved sheet. The problem treated in the present paper, which is a part of that series, is the determination of the critical shear stress of a cylindrically curved rectangular panel. For panels having simply supported edges this problem is solved theoretically (see appendix), and computed curves are provided for finding the critical shear stress. Estimated results are also given for curved rectangular panels having clamped edges.

RESULTS AND DISCUSSION

The critical shear stress τ_{cr} for cylindrically curved panels is given by the equation

$$\tau_{cr} = k_s \frac{\pi^2 D}{b^2 t}$$

where

k_s critical-shear-stress coefficient, established by geometry of panel and type of edge support

D flexural stiffness of panel per unit length $\left(\frac{Et^3}{12(1 - \mu^2)}\right)$

E Young's modulus of elasticity

μ Poisson's ratio

b axial or circumferential dimension of panel, whichever is smaller (except where noted)

t thickness of panel

Two charts are presented, one for panels having curved sides longer than the straight sides and the other for panels having straight sides longer than the curved sides. In each of these charts the critical-shear-stress coefficient k_s is plotted against a curvature parameter Z defined by the equation

$$Z = \frac{b^2}{rt}\sqrt{1 - \mu^2}$$

or

$$Z = \left(\frac{b}{r}\right)^2 \frac{r}{t}\sqrt{1 - \mu^2}$$

where

r radius of curvature of panel

Panels with simply supported edges.- The critical-shear-stress coefficients k_s for curved panels with simply supported edges are given in figures 1 and 2. In figure 1, the critical-shear-stress coefficients are given for panels long in the circumferential direction; b is measured axially with the result that the values of k_s and Z are defined in the manner appropriate to a cylinder. In figure 2, the critical-shear-stress coefficients are given for panels long in the axial direction; b is measured circumferentially so that the values of k and Z are defined in the manner appropriate to an infinitely long curved strip. The solid curves in each figure are computed, and the dashed curve in figure 2 is estimated by a method described in the section entitled "Estimation of Critical Stress."

With regard to displacements in the median surface during buckling, the boundary conditions for which the curves of figures 1 and 2 apply are zero displacement along each edge and unrestrained motion normal to each edge. The available evidence indicates that if both normal and tangential motion were completely restrained, the curves of figure 1 would be raised only slightly, but the curves of figure 2 might be raised considerably at intermediate values of Z for large values of panel length-width ratio a/b. (See section on critical shear stress of curved panel in reference 1.)

In figure 1 all the buckling curves for panels of fixed length-width ratio approach the curves for a complete cylinder at high values of Z. The same trend exists when the axial length is greater than the circumferential length, as the curves of figure 2 suggest by approaching the slope 0.75 appropriate to cylinder curves at high values of Z. This result can be explained by the fact that geometrically a curved panel of any given length-width ratio approaches a cylinder as the curvature increases. Panels having a large ratio of circumferential length to axial length approach a cylinder at lower values of Z (defined as in fig. 1) than panels having a small ratio of circumferential length to axial length.

Estimation of critical stress.- A comparison of the computed results for the critical stress coefficients of simply supported curved panels (figs. 1 and 2) with results for certain known limiting cases indicates the possibility of making reasonable estimates for panels of other length-width ratios and edge-support conditions. The known limiting cases used in this comparison are the critical stress coefficients for the flat panel $(Z = 0)$, the complete cylinder, and the infinitely long curved strip. (See fig. 3.) Figure 4 shows the comparison for panels long in the circumferential direction. In order to permit the comparison, the curve for the strip was plotted by using the same parameters as were used in the other curves; that is, k_s and Z were defined in terms of the axial rather than the circumferential dimension of the panel (dimension b_1 in fig. 3). Figure 5 shows the comparison for panels long in the axial direction. The cylinder curve is replotted in terms of dimension b_2 in figure 3 so that the same parameters are used as were used for the other curves.

In each of figures 4 and 5 the first three panel buckling curves were computed and the fourth curve was estimated. These estimated curves were obtained by using the known limiting results as guides and by extrapolating the trends observed in the cases from which computed results were available.

Panels with clamped edges.- Figures 6 and 7 give estimated theoretical critical-shear-stress coefficients for curved panels with clamped edges. The estimates were made in the aforementioned manner by making use of the known shear-stress coefficients for cylinders and for long curved strips with clamped edges (references 2 and 3, respectively) and available results for flat panels (references 4 and 5) and by extrapolating from the known results for simply supported panels. With respect to edge displacements in the median surface during buckling, the boundary conditions for which the curves of figures 5 and 6 apply are zero displacement normal to each edge, and unrestrained motion along each edge. The available evidence indicates that complete restraint of both normal and tangential motion would affect the curves of figures 5 and 6 only slightly. (See discussion of boundary conditions in reference 1.)

Experimental Verification

A study of references 6 to 10, which contain test data on the critical shear stress for curved sheet, revealed that in the various investigations different types of test specimens (fig. 8) and also different methods of defining the experimental critical stress were used. Because of the different types of test specimens and different methods used, a wide range of values for the critical stresses must be expected. In order to make the comparison between theory and experiment, therefore, the test data were divided into two groups according to whether the buckling load would be appreciably affected by initial eccentricities (fig. 9). The theoretical curves used were those for simply supported curved panels.

Experimental buckling stresses only slightly affected by initial eccentricities.- With but three exceptions, the experimental critical stresses of Rafel (reference 6) and of Rafel and Sandlin (reference 7) correspond to snap buckling (fig. 9(a)). With any appreciable initial eccentricities, deflections tend to increase gradually with load and no snap buckle occurs; this fact indicates that the initial eccentricities in the test specimens are small. The three specimens in which snap buckling did not occur fell in the range of $6<Z<12$, and the buckling stress was taken to correspond to the load at which the compressive diagonal stress on one side of the sheet ceased to increase with increasing load. The critical loads for the Moore and Wescoat data were obtained from the loads corresponding to the top of the knee of the torque-twist curves (reference 8). The torque-twist curves represent averages of the behavior of all the panels in the cylinders of reference 8 and are thus relatively insensitive to local imperfections. Figure 9(a) indicates that buckling stresses defined in such a way as to be rather insensitive to local imperfections are in good agreement with, but slightly lower than, the theoretical critical stresses for curved panels with simply supported edges.

Experimental buckling stresses considerably affected by initial eccentricities.- In the tests by Chiarito (reference 9) the critical stress is defined as the stress at which wrinkling first becomes visually perceptible in any panel of a multipanel specimen. Except for one snap buckle, the critical stresses in the tests by Kuhn and Levin (reference 10) were defined by the point of first departure from a straight line of the curve obtained by plotting the readings of a diagonally mounted Tuckerman optical strain gage against the applied load. In both of these methods initial eccentricities of the test specimens can be expected to result in relatively low experimental critical stresses.

A comparison between the theoretical critical stresses for curved panels and the experimental critical stresses of Chiarito and of Kuhn and Levin is made in figure 9(b). As might be expected because of sensitivity to local imperfections, the experimental data are, in general, considerably below the theoretical curves. The data of Kuhn and Levin for $\frac{a}{b} = 3$ at large values of Z, however, are appreciably above the theoretical curves. One possible explanation for this rather surprising behavior is that all specimens for $\frac{a}{b} = 3$ had two intermediate bulkheads which were not attached to the sheet but may have provided additional restraint against buckling at high curvatures by preventing inward displacement of the sheet on buckling.

CONCLUDING REMARKS

The critical shear stresses given by a theoretical solution based on small-deflection theory for simply supported curved rectangular panels were found to be in good agreement with experimental critical stresses defined in such a way as to be rather insensitive to local imperfections. A method is suggested for estimating the critical shear stresses for curved panels having length-width ratios and types of edge support different from those for which computed results are available.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., March 11, 1947

APPENDIX

THEORETICAL SOLUTION

Symbols

a — axial or circumferential dimension of panel, whichever is larger

b — axial or circumferential dimension of panel, whichever is smaller

m, n, p, q — integers

r — radius of curvature of panel

t — thickness of panel

u — displacement of points on median surface of panel in axial (x-) direction

v — displacement of points on median surface of panel in circumferential (y-) direction

w — displacement of points on median surface in radial direction; positive outward

x — axial coordinate of panel

y — circumferential coordinate of panel

D — flexural stiffness of panel per unit length $\left(\frac{Et^3}{12(1-\mu^2)}\right)$

E — Young's modulus of elasticity

F — Airy's stress function for median-surface stresses produced by buckle deformation $\left(-\frac{\partial^2 F}{\partial x\,\partial y}\right.$, shear stress; $\frac{\partial^2 F}{\partial x^2}$, compressive stress in y-direction; $\left.\frac{\partial^2 F}{\partial y^2}\text{, compressive stress in x-direction}\right)$

Z curvature parameter $\left(\frac{b^2}{rt}\sqrt{1 - \mu^2}\right)$

a_{mn} coefficients of terms in deflection functions

k_s critical shear-stress coefficient appearing in formula $\tau_{cr} = \frac{k_s \pi^2 D}{b^2 t}$

$$M_{pq} = \frac{\pi^2}{32\beta^3 k_s}\left[(p^2 + q^2\beta^2)^2 + \frac{12\beta^4 p^4 Z^2}{\pi^4(p^2 + q^2\beta^2)^2}\right]$$

$$N_{pq} = \frac{\pi^2}{32\beta^3 k_s}\left[(p^2\beta^2 + q^2)^2 + \frac{12\beta^8 p^4 Z^2}{\pi^4(p^2\beta^2 + q^2)^2}\right]$$

$\beta = \frac{a}{b}$

μ Poisson's ratio

τ_{cr} critical shear stress

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$

∇^{-4} = inverse of ∇^4 defined by $\nabla^{-4}(\nabla^4 w) = w$

Method of Solution

Equation of equilibrium. - The critical shear stress which causes a curved rectangular panel to buckle may be obtained by solving the following equation of equilibrium (reference 1).

$$D\nabla^4 w + \frac{Et}{r^2}\nabla^{-4}\frac{\partial^4 w}{\partial x^4} + 2\tau_{cr} t \frac{\partial^2 w}{\partial x\, \partial y} = 0 \qquad (1)$$

where x and y are, respectively, the axial and circumferential coordinates. Division of equation (1) by D gives

$$\nabla^4 w + \frac{12Z^2}{b^4}\nabla^{-4}\frac{\partial^4 w}{\partial x^4} + 2k_s\frac{\pi^2}{b^2}\frac{\partial^2 w}{\partial x\, \partial y} = 0 \qquad (2)$$

The equation of equilibrium may be represented by

$$Qw = 0 \tag{3}$$

where Q is the operator defined by

$$Q = \nabla^4 + \frac{12Z^2}{b^4}\nabla^{-4}\frac{\partial^4}{\partial x^4} + 2k_s\frac{\pi^2}{b^2}\frac{\partial^2}{\partial x\,\partial y} \tag{4}$$

Solution for panels having axial length greater than circumferential length.- Equation (3) may be solved by the Galerkin method as outlined in references 1 and 11. As suggested in reference 1 for simply supported curved rectangular panels, the following series expansion is used for w:

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{5}$$

The coordinate system used is shown in figure 10(a). The coefficients a_{mn} are then chosen to satisfy the equations

$$\int_0^a\int_0^b \sin\frac{p\pi x}{a}\sin\frac{q\pi y}{b}\,Qw\,dx\,dy = 0 \tag{6}$$

where

$$p = 1,2, \ldots \text{ and } q = 1,2, \ldots$$

When the operations indicated in equation (6) are performed, a set of homogeneous linear algebraic equations in the unknown coefficients a_{pq} is obtained with k_s appearing as a parameter. The solution for the critical-shear-stress coefficient k_s is then found as the minimum value of k_s for which the algebraic equations have a nonvanishing solution for the unknowns a_{pq}.

The boundary conditions at each edge that are implied by this method of solution are zero radial deflection and edge moment, no displacement along the edge, and free displacement normal to the edge (see reference 12); that is,

$$\left.\begin{aligned}
&w(0,y) = w(a,y) = 0 && w(x,0) = w(x,b) = 0 \\
&\frac{\partial^2 w}{\partial x^2}(0,y) = \frac{\partial^2 w}{\partial x^2}(a,y) = 0 && \frac{\partial^2 w}{\partial y^2}(x,0) = \frac{\partial^2 w}{\partial y^2}(x,b) = 0 \\
&v(0,y) = v(a,y) = 0 && u(x,0) = u(x,b) = 0 \\
&\frac{\partial^2 F}{\partial y^2}(0,y) = \frac{\partial^2 F}{\partial y^2}(a,y) = 0 && \frac{\partial^2 F}{\partial x^2}(x,0) = \frac{\partial^2 F}{\partial x^2}(x,b) = 0
\end{aligned}\right\} \quad (7)$$

Substitution of the values of Q and w given by equations (4) and (5), respectively, into equation (6) leads to the following set of algebraic equations:

$$a_{pq}\left[\left(p^2 + q^2\beta^2\right)^2 + \frac{12\beta^4 p^4 Z^2}{\pi^4\left(p^2 + q^2\beta^2\right)^2}\right] + \frac{32\beta^3 k_s}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\frac{mnpq}{\left(m^2 - p^2\right)\left(n^2 - q^2\right)} = 0 \quad (8)$$

where $p = 1,2, \ldots$; $q = 1,2, \ldots$; and the summation includes only those values of m and n for which $m \pm p$ and $n \pm q$ are odd. The condition for a nonvanishing solution of these equations is the vanishing of the determinant of the coefficients of the unknowns a_{mn}. This infinite determinant can be factored into the product of two infinite subdeterminants, one in which $p \pm q$ is even, and one in which $p \pm q$ is odd. The vanishing of these subdeterminants leads to the following determinantal equations. The equation in which $p \pm q$ is even is

$$
\begin{array}{r}
 \\
p=1,\ q=1 \\
p=1,\ q=3 \\
p=2,\ q=2 \\
p=3,\ q=1 \\
p=1,\ q=5 \\
p=2,\ q=4 \\
p=3,\ q=3 \\
p=1,\ q=7 \\
p=2,\ q=6 \\
p=3,\ q=5 \\
\cdot \\
\cdot \\
\cdot
\end{array}
\begin{array}{cccccccccccc}
a_{11} & a_{13} & a_{22} & a_{31} & a_{15} & a_{24} & a_{33} & a_{17} & a_{26} & a_{35} & \cdots
\end{array}
$$

$$
\left|
\begin{array}{ccccccccccc}
M_{11} & 0 & \frac{4}{9} & 0 & 0 & \frac{8}{45} & 0 & 0 & \frac{4}{35} & 0 & \cdots \\
0 & M_{13} & -\frac{4}{5} & 0 & 0 & \frac{8}{7} & 0 & 0 & \frac{4}{9} & 0 & \cdots \\
\frac{4}{9} & -\frac{4}{5} & M_{22} & -\frac{4}{5} & -\frac{20}{63} & 0 & \frac{36}{25} & -\frac{28}{135} & 0 & \frac{4}{7} & \cdots \\
0 & 0 & -\frac{4}{5} & M_{31} & 0 & -\frac{8}{25} & 0 & 0 & -\frac{36}{175} & 0 & \cdots \\
0 & 0 & -\frac{20}{63} & 0 & M_{15} & -\frac{40}{27} & 0 & 0 & \frac{20}{11} & 0 & \cdots \\
\frac{8}{45} & \frac{8}{7} & 0 & -\frac{8}{25} & -\frac{40}{27} & M_{24} & -\frac{72}{35} & -\frac{56}{99} & 0 & \frac{8}{3} & \cdots \\
0 & 0 & \frac{36}{25} & 0 & 0 & -\frac{72}{35} & M_{33} & 0 & -\frac{108}{125} & 0 & \cdots \\
0 & 0 & -\frac{28}{135} & 0 & 0 & -\frac{56}{99} & 0 & M_{17} & -\frac{28}{13} & 0 & \cdots \\
\frac{4}{35} & \frac{4}{9} & 0 & -\frac{36}{175} & \frac{20}{11} & 0 & -\frac{108}{125} & -\frac{28}{13} & M_{26} & -\frac{36}{11} & \cdots \\
0 & 0 & \frac{4}{7} & 0 & 0 & \frac{8}{3} & 0 & 0 & -\frac{36}{11} & M_{35} & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot &
\end{array}
\right| = 0 \qquad (9)
$$

and the equation in which $p \pm q$ is odd is

$$
\begin{array}{r}
\\
p=1,\ q=2 \\
p=2,\ q=1 \\
p=1,\ q=4 \\
p=2,\ q=3 \\
p=3,\ q=2 \\
p=1,\ q=6 \\
p=2,\ q=5 \\
p=3,\ q=4 \\
p=1,\ q=8 \\
p=2,\ q=7 \\
\cdot \\
\cdot \\
\cdot
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
a_{12} & a_{21} & a_{14} & a_{23} & a_{32} & a_{16} & a_{25} & a_{34} & a_{18} & a_{27} & \cdots
\end{array} \\
\left|
\begin{array}{cccccccccccc}
M_{12} & -\frac{4}{9} & 0 & \frac{4}{5} & 0 & 0 & \frac{20}{63} & 0 & 0 & \frac{28}{135} & \cdots \\
-\frac{4}{9} & M_{21} & -\frac{8}{45} & 0 & \frac{4}{5} & -\frac{4}{35} & 0 & \frac{8}{25} & -\frac{16}{189} & 0 & \cdots \\
0 & -\frac{8}{45} & M_{14} & -\frac{8}{7} & 0 & 0 & \frac{40}{27} & 0 & 0 & \frac{56}{99} & \cdots \\
\frac{4}{5} & 0 & -\frac{8}{7} & M_{23} & -\frac{36}{25} & -\frac{4}{9} & 0 & \frac{72}{35} & -\frac{16}{55} & 0 & \cdots \\
0 & \frac{4}{5} & 0 & -\frac{36}{25} & M_{32} & 0 & -\frac{4}{7} & 0 & 0 & -\frac{28}{75} & \cdots \\
0 & -\frac{4}{35} & 0 & -\frac{4}{9} & 0 & M_{16} & -\frac{20}{11} & 0 & 0 & \frac{28}{13} & \cdots \\
\frac{20}{63} & 0 & \frac{40}{27} & 0 & -\frac{4}{7} & -\frac{20}{11} & M_{25} & -\frac{8}{3} & -\frac{80}{117} & 0 & \cdots \\
0 & \frac{8}{25} & 0 & \frac{72}{35} & 0 & 0 & -\frac{8}{3} & M_{34} & 0 & -\frac{56}{55} & \cdots \\
0 & -\frac{16}{189} & 0 & -\frac{16}{55} & 0 & 0 & -\frac{80}{117} & 0 & M_{18} & -\frac{112}{45} & \cdots \\
\frac{28}{135} & 0 & \frac{56}{99} & 0 & -\frac{28}{75} & \frac{28}{13} & 0 & \frac{56}{55} & -\frac{112}{45} & M_{27} & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot &
\end{array}
\right| = 0
\end{array}
\qquad (10)
$$

where

$$
M_{pq} = \frac{\pi^2}{32\beta^3 k_s}\left[\left(p^2 + q^2\beta^2\right)^2 + \frac{12\beta^4 p^4 Z^2}{\pi^4\left(p^2 + q^2\beta^2\right)^2}\right]
$$

These determinants give the buckling loads of curved panels with various length-width ratios $(\beta \geqq 1)$ for buckle patterns respectively symmetrical and antisymmetrical about the center of the panel.

By use of a finite determinant including the rows and columns corresponding to the most important terms in the expansion for w (equation (5)), equations (9) and (10) were solved by a matrix iteration method (reference 13) for the lowest value of k_s which satisfied those equations. The lower of the two values of k_s found by solving equations (9) and (10) is the critical-shear-stress coefficient for the particular values of β and Z under consideration. The curve of critical-shear-stress coefficient against Z for a given value of β computed in this manner shows cusps; however, as the precise location of all the cusps involves a prohibitive amount of labor without any significant increase in accuracy, the cusps were faired out in figure 2. Table 1 presents the relative magnitudes of the coefficients of the terms used in the solutions, and table 2 gives the computed stress coefficients.

<u>Solution for panels having circumferential length greater than axial length</u>.- When the circumferential dimension is greater than the axial dimension, a and b can be interchanged in equation (5) in order to retain b as the shorter dimension, as follows:

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{b} \sin \frac{n\pi y}{a} \tag{11}$$

The coordinate system used is shown in figure 10(b). This problem is solved in a manner similar to that used in solving the problem involving axial dimension greater than circumferential dimension. The set of algebraic equations for the unknown Fourier coefficients is now

$$\begin{aligned} a_{pq} & \left[\left(p^2\beta^2 + q^2\right)^2 + \frac{12\beta^8 p^4 Z^2}{\pi^4 \left(p^2\beta^2 + q^2\right)^2} \right] \\ & + \frac{32\beta^3 k_s}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \frac{mnpq}{\left(m^2 - p^2\right)\left(n^2 - q^2\right)} = 0 \end{aligned} \tag{12}$$

where $m \pm p$ and $n \pm q$ must be odd for a_{mn} to have a value. The condition for a nonvanishing solution of these equations is the vanishing of the determinant of the coefficients of the unknowns a_{mn}.

This determinant can be factored into two infinite subdeterminants which are identical to those in equations (9) and (10) except for the diagonal terms. In this case, each diagonal term M_{pq} is replaced by

$$N_{pq} = \frac{\pi^2}{32\beta^3 k_s}\left[(p^2\beta^2 + q^2)^2 + \frac{12\beta^8 p^4 Z^2}{\pi^4(p^2\beta^2 + q^2)^2}\right]$$

The conditions for the vanishing of these determinants were found in the same manner as that used in solving equations (9) and (10) for buckling loads of panels with various length-width ratios for buckle patterns symmetrical or antisymmetrical about the center of the panel. The lower of the two values of k_s is the critical-shear-stress coefficient for the particular values of β and Z under consideration. The relative magnitudes of the coefficients of the terms used and the computed stress coefficients are presented in tables 1 and 2, respectively. Figure 1 shows the critical-shear-stress coefficients for simply supported curved panels having the circumferential length greater than the axial length; the curves were faired in a manner similar to that for the curves of figure 2.

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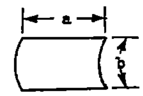
TABLE I

RELATIVE MAGNITUDES OF DEFLECTION-FUNCTION COEFFICIENTS USED IN SOLUTION



Even determinant

$\frac{a}{b}$	z	a_{11}	a_{13}	a_{22}	a_{31}	a_{15}	a_{24}	a_{33}	a_{42}	a_{51}	a_{17}	a_{26}	a_{35}	a_{44}	a_{53}	a_{62}	a_{28}	a_{37}	a_{46}	a_{55}	a_{48}	a_{57}
1	1	1	-0.07	-0.30	-0.07	-0.005	-0.004	0.04	-0.002	----	----	----	0.004	-0.003	----	----	----	----	----	----	----	----
	10	1	-.13	.43	-.12	-.007	-.005	.07	-.007	----	----	-.0005	.008	----	----	----	----	----	----	----	----	----
	30	-.75	.57	1	.26	----	-.17	-.36	-.10	----	----	-.02	-.004	.04	----	----	----	----	----	----	----	----
	100	-.02	1	-.37	----	-.13	.65	-.39	----	----	----	.003	.18	----	----	----	----	----	0.02	----	----	----
	1000	----	.09	----	----	1	-.10	----	----	----	-.48	.95	-.17	----	----	----	-0.24	0.47	-.08	----	0.11	----
1.5	1	1	-.04	.30	-.15	----	----	.04	.007	-.01	----	----	----	.002	0.007	-0.0006	----	----	----	----	----	----
	10	1	-.06	.39	-.17	----	.002	.06	.006	-.02	----	----	----	.004	.01	----	----	----	----	----	----	----
	30	1	-.28	-.99	-.29	----	.07	.34	.14	----	----	----	.01	-.04	.02	----	----	----	----	----	----	----
	100	-.16	1	.95	.05	-.06	-.72	-.95	-.15	----	----	----	.19	.32	----	----	----	----	----	-0.04	----	----
	1000	----	-.06	----	----	.94	-.40	----	----	----	-.28	1	-.47	----	----	----	-.11	.50	-.23	----	.11	----
2	1	1	-.03	.34	-.33	----	.006	.05	-.04	-.01	----	----	.003	.001	----	----	----	----	----	----	----	----
	10	1	-.04	.39	-.28	----	.004	.06	-.05	-.01	----	----	.004	.003	----	----	----	----	----	----	----	----
	30	1	-.13	.71	-.21	----	-.02	.20	-.10	-.01	----	----	.03	.02	----	----	----	----	----	----	----	----
	100	.32	-.52	1	-.06	----	-.34	.78	-.16	----	----	----	-.06	.24	----	----	----	----	----	.04	----	----
	1000	----	1	-.02	----	-.50	.88	-.05	----	----	----	-.32	.53	----	----	----	----	-.10	.22	----	----	0.06

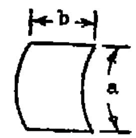
Odd determinant

$\frac{a}{b}$	z	a_{12}	a_{21}	a_{14}	a_{23}	a_{32}	a_{41}	a_{16}	a_{25}	a_{34}	a_{43}	a_{52}	a_{27}	a_{36}	a_{45}	a_{54}	a_{47}	a_{18}	a_{110}	a_{29}	a_{38}
1	1	1	1	-0.01	-0.28	-0.28	-0.01	----	-0.02	0.01	0.01	-0.02	----	----	----	----	----	----	----	----	----
	10	1	.83	-.02	-.31	-.28	-.01	----	.02	.02	.02	-.02	----	----	----	----	----	----	----	----	----
	30	1	.37	-.08	-.46	-.26	-.01	-0.007	-.01	.08	.03	----	----	----	----	----	----	----	----	----	----
	100	1	.05	-.44	-.91	-.20	----	-.01	.16	.39	.10	----	----	----	-0.07	----	----	----	----	----	----
	1000	----	----	-.26	----	----	----	1	.54	.01	----	----	-0.92	-0.47	----	----	0.18	0.24	----	0.08	0.37
1.5	1	-.46	1	----	-.12	.30	-.06	----	-.008	-.002	-.03	.02	----	----	----	0.003	----	----	----	----	----
	10	-.61	1	----	-.20	.41	-.09	----	-.01	-.01	.05	.01	----	----	----	.004	----	----	----	----	----
	30	1	.56	-.07	-.56	-.62	-.11	----	-.01	.11	.16	.01	----	----	----	----	----	----	----	----	----
	100	1	.05	-.16	-.68	-.17	.06	----	.04	.25	.08	----	----	----	-.05	----	----	----	----	----	----
	1000	.04	----	1	-.07	----	----	-.39	.90	-.13	----	----	-.19	.46	-.07	----	.12	----	----	----	----
2	1	-.28	1	-.004	-.07	.30	-.12	----	-.005	.005	.03	.01	----	----	----	----	----	----	----	----	----
	10	-.38	1	-.003	-.13	.44	-.18	----	-.008	-.001	.06	-.003	----	----	----	----	----	----	----	----	----
	30	-.79	-.79	.03	.52	1	.26	----	.02	-.11	-.30	-.10	----	----	----	----	----	----	----	----	----
	100	1	-.08	-.11	.66	-.26	----	----	-.02	.22	-.12	----	----	----	.03	-.003	----	----	----	----	----
	1000	.06	----	1	-.17	----	----	-.25	.99	-.29	----	----	-.12	.51	-.16	----	.13	----	----	----	----

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - Concluded

RELATIVE MAGNITUDES OF DEFLECTION-FUNCTION COEFFICIENTS USED IN SOLUTION - Concluded



Even determinant

$\frac{a}{b}$	Z	a_{11}	a_{13}	a_{22}	a_{31}	a_{15}	a_{24}	a_{33}	a_{42}	a_{51}	a_{17}	a_{26}	a_{35}	a_{44}	a_{37}
1.5	1	1	-0.16	0.31	-0.04	-0.01	-0.006	0.04	-----	----	-----	-0.001	0.006	0.003	----
	10	1	-.85	.89	-.14	-.003	-.22	.23	-----	----	-----	-.02	-.001	.01	----
	30	-.06	1	-.44	.04	-.12	.48	-.26	0.02	----	-----	.02	.08	-.03	----
	100	-.01	-.83	-.04	----	1	.93	.14	-----	----	-0.07	-.48	-.43	-----	0.06
2	1	1	-.35	.36	-.03	-.008	-.05	.05	.005	----	-----	-----	.003	.001	----
	10	-.25	1	.50	.04	-.10	-.36	-.16	.0004	----	-----	-----	.04	.004	----

Odd determinant

$\frac{a}{b}$	Z	a_{12}	a_{21}	a_{14}	a_{23}	a_{32}	a_{41}	a_{16}	a_{25}	a_{34}	a_{43}	a_{52}	a_{36}	a_{18}	a_{27}	a_{38}
1.5	1	1	-0.45	-0.06	0.30	-0.12	-0.004	----	0.02	0.03	0.002	-0.008	-----	-----	----	---
	10	1	-.36	-.10	.37	-.13	-.004	----	.01	.05	-.001	-.009	-----	-----	----	---
	30	1	-.14	-.54	.81	-.22	-.002	----	-.14	.24	-.04	-.02	-----	-----	----	---
	100	.007	----	1	-.24	-----	-----	-0.27	.68	-.24	-----	-----	0.18	-0.03	-0.02	0.03
2	1	1	-.28	-.12	.30	-.07	-.004	----	.01	.03	.005	-.005	-----	-----	----	---
	10	1	-.23	-.33	.50	-.12	-.004	----	-.04	.09	.0001	-.007	-----	-----	----	---

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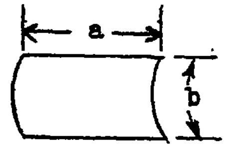
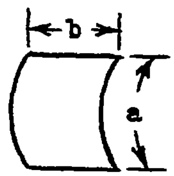
COMMITTEE FOR AERONAUTICS

TABLE 2.- THEORETICAL CRITICAL-SHEAR-STRESS COEFFICIENTS



$\frac{a}{b}$	Z	k_s	
		Even determinant	Odd determinant
1	0	9.35	------
	1	9.44	11.59
	10	11.65	12.77
	30	18.57	17.59
	100	34.65	33.55
	1000	157.4	164.5
1.5	0	7.07	7.97
	1	7.12	8.03
	10	8.55	9.75
	30	14.30	15.38
	100	30.54	27.15
	1000	136.6	129.7
2	1	6.62	6.65
	10	7.65	8.43
	30	12.48	14.29
	100	26.96	26.19
	1000	117.3	118.9



$\frac{a}{b}$	Z	k_s	
		Even determinant	Odd determinant
1.5	1	7.37	7.99
	10	10.38	9.49
	30	15.23	15.51
	100	32.24	30.73
2	1	6.68	6.61
	10	8.98	8.95

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

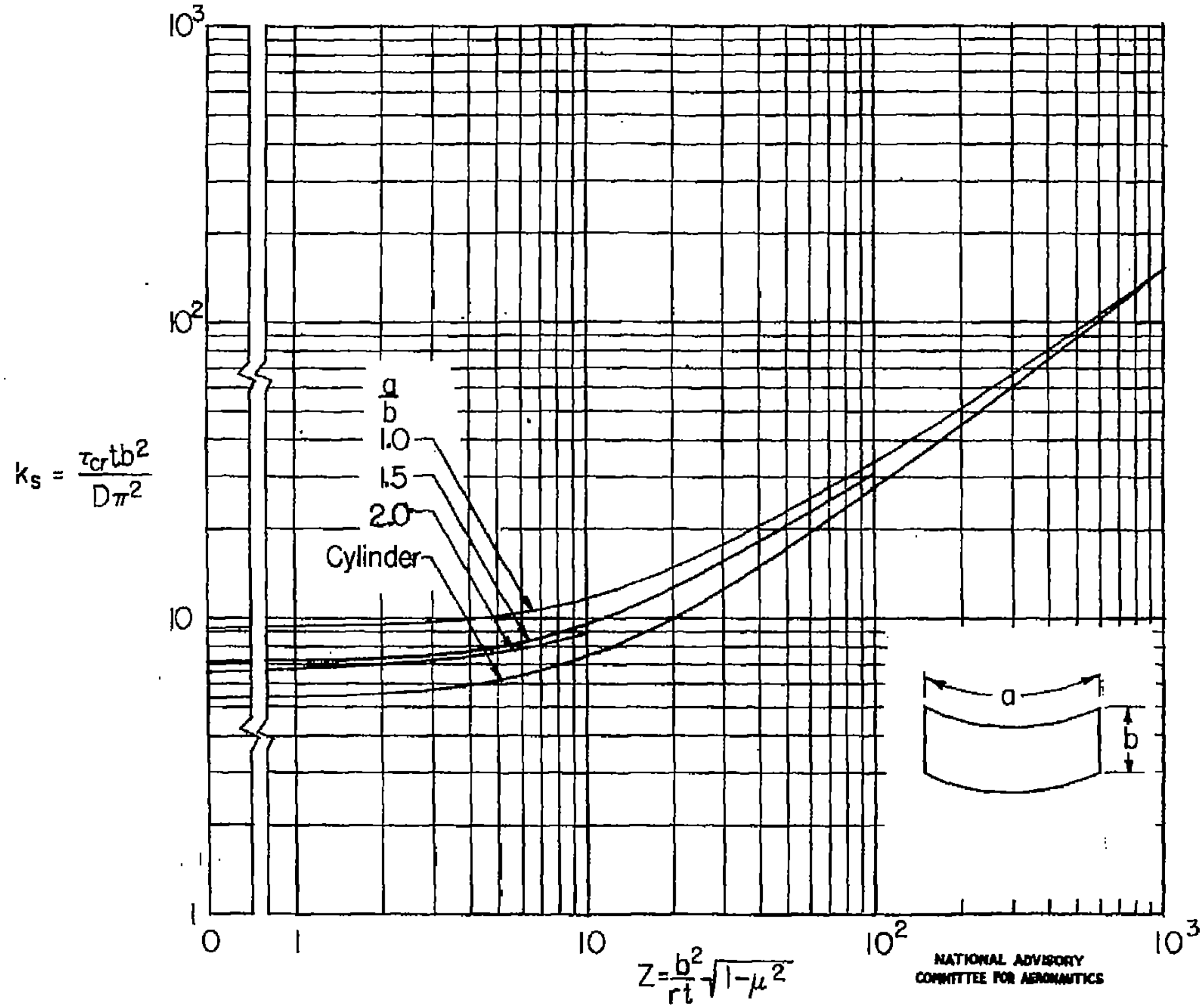


Figure 1.- Critical-shear-stress coefficients for simply supported curved panels having circumferential length greater than axial length.

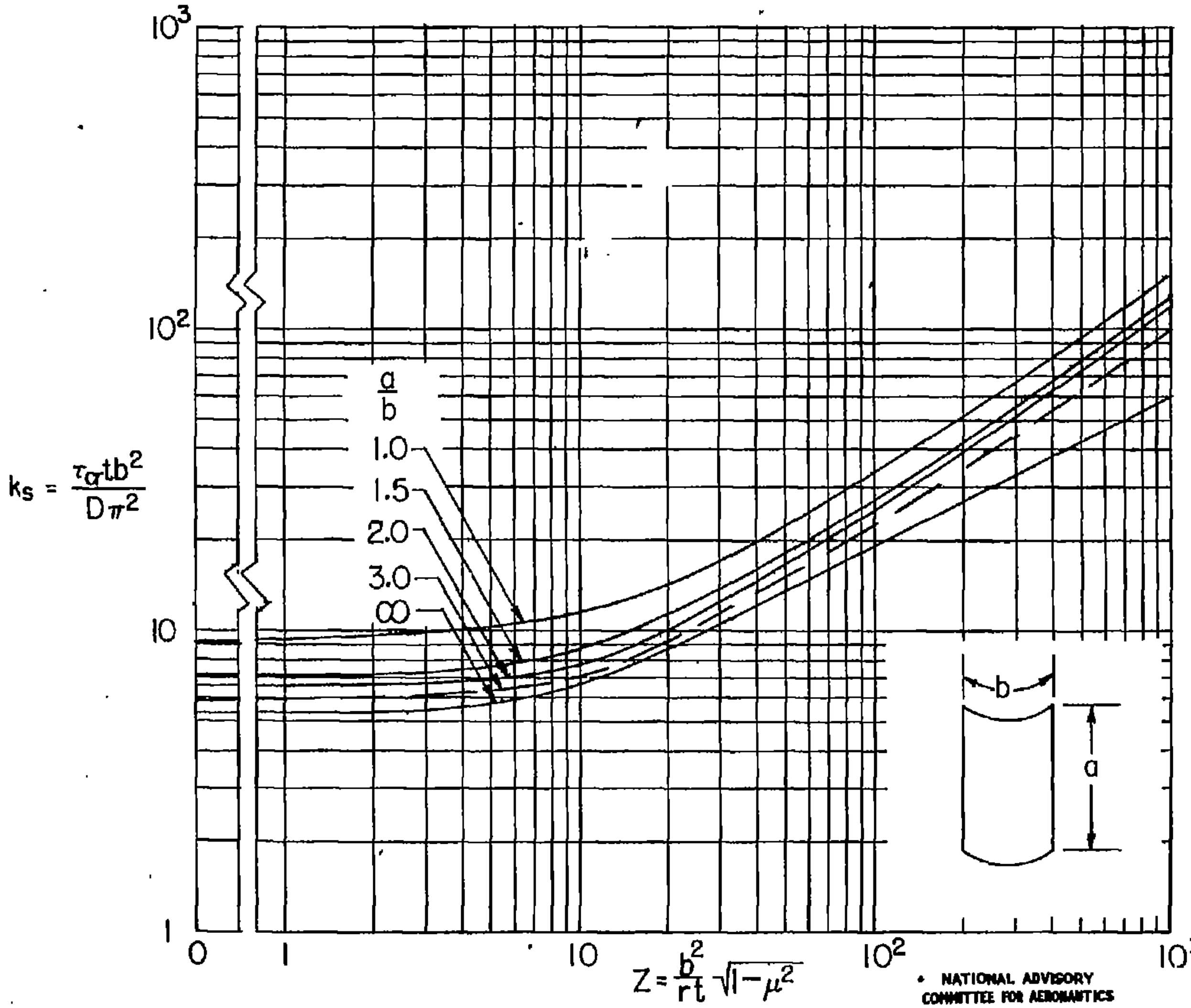


Figure 2.- Critical-shear-stress coefficients for simply supported curved panels having axial length greater than circumferential length.

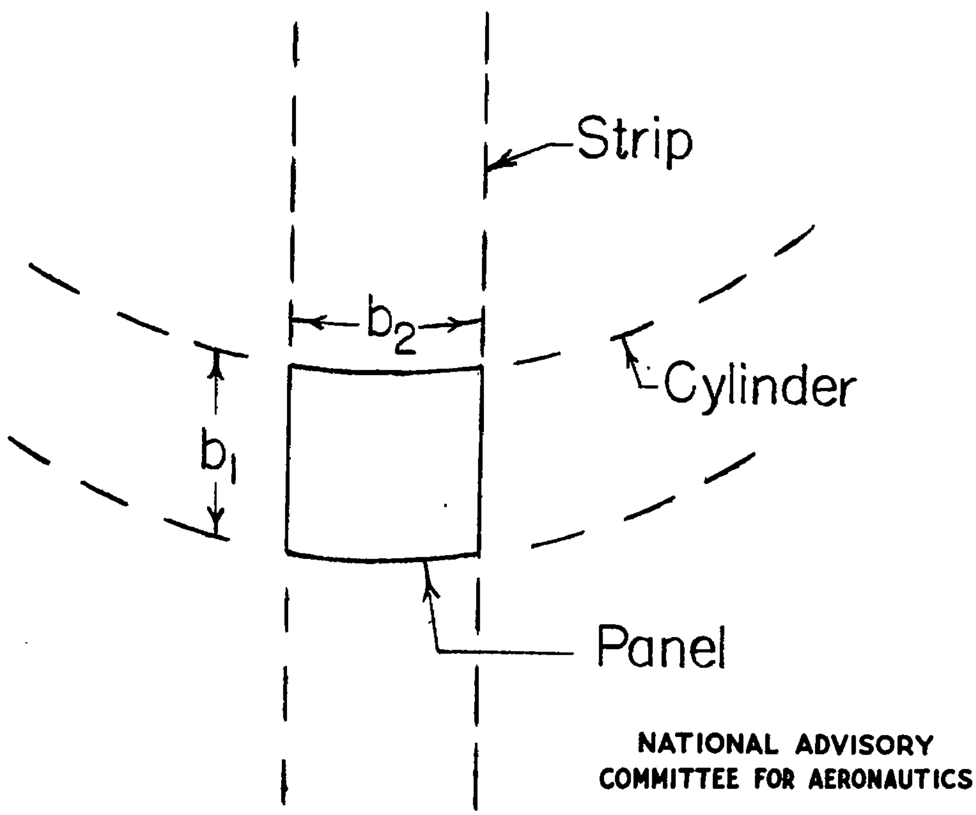


Figure 3.- Curved rectangular panel. Limiting cases for a complete cylinder and an infinitely long strip also shown.

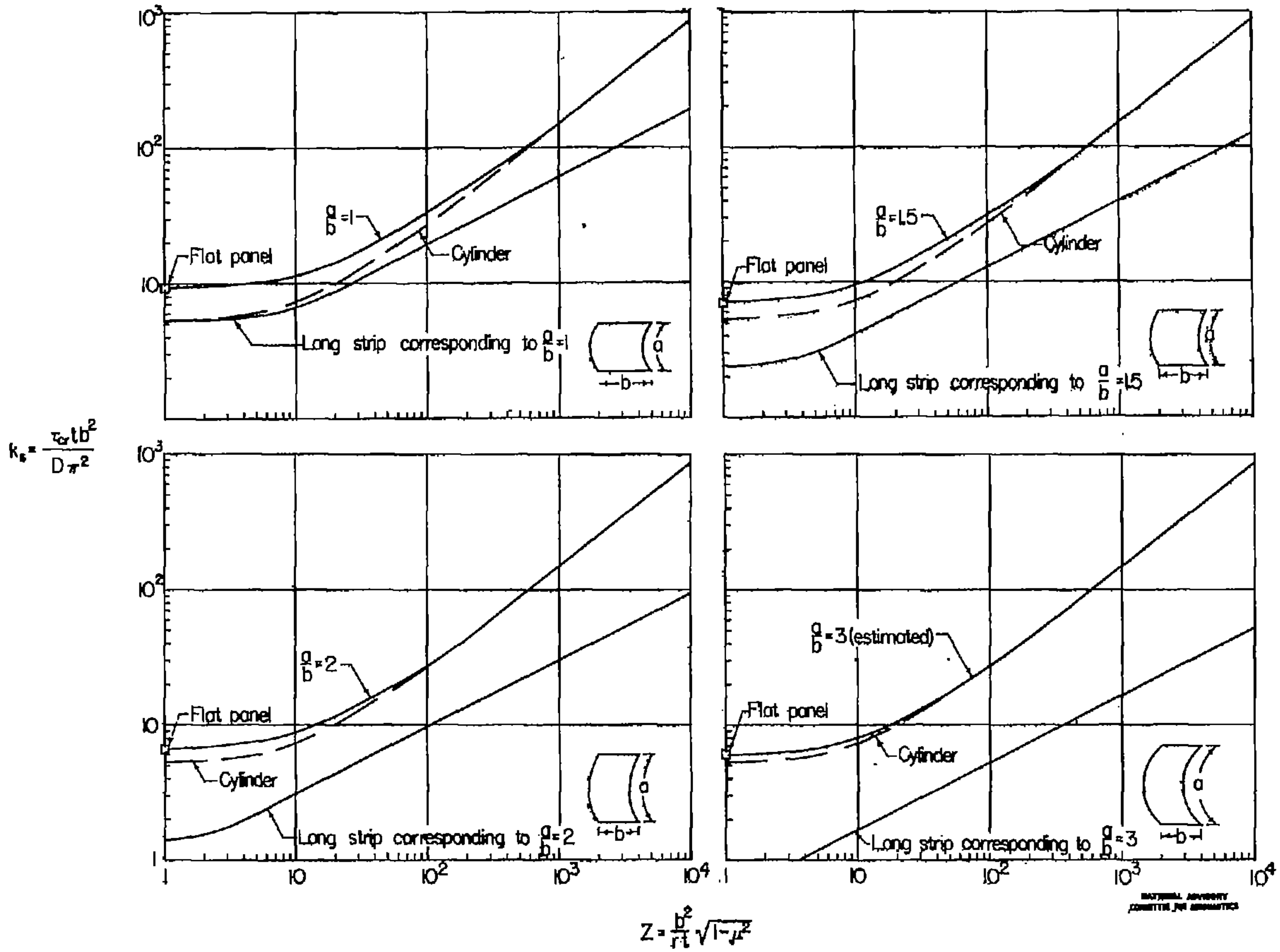


Figure 4.- Critical-shear-stress coefficients for simply supported curved panels compared with those for cylinders and long curved strips (k_s and Z are defined in terms of axial length of panel).

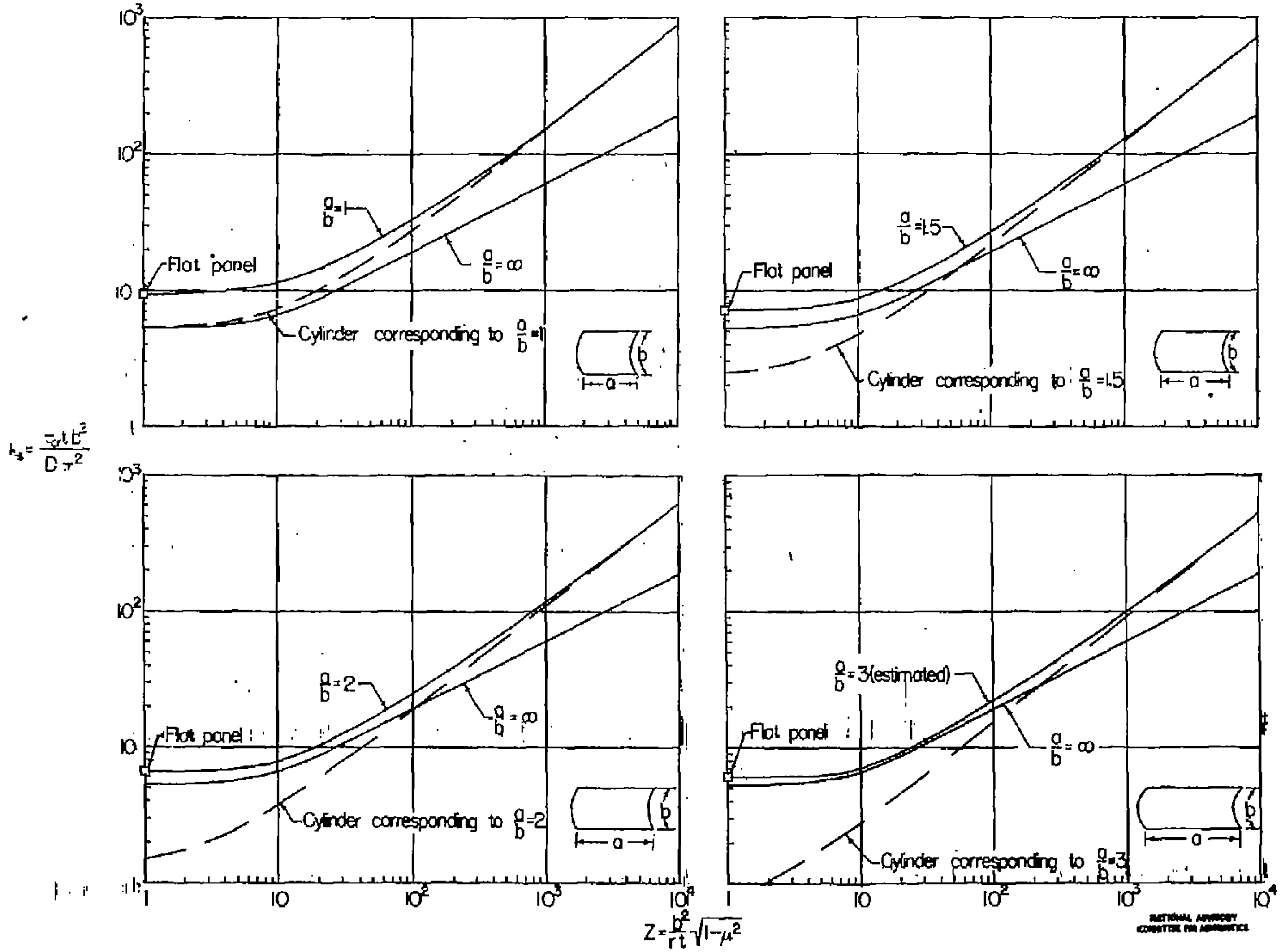


Figure 5.- Critical-shear-stress coefficients for simply supported curved panels compared with those for cylinders and infinitely long curved strips (k_s and Z are defined in terms of circumferential length of panel).

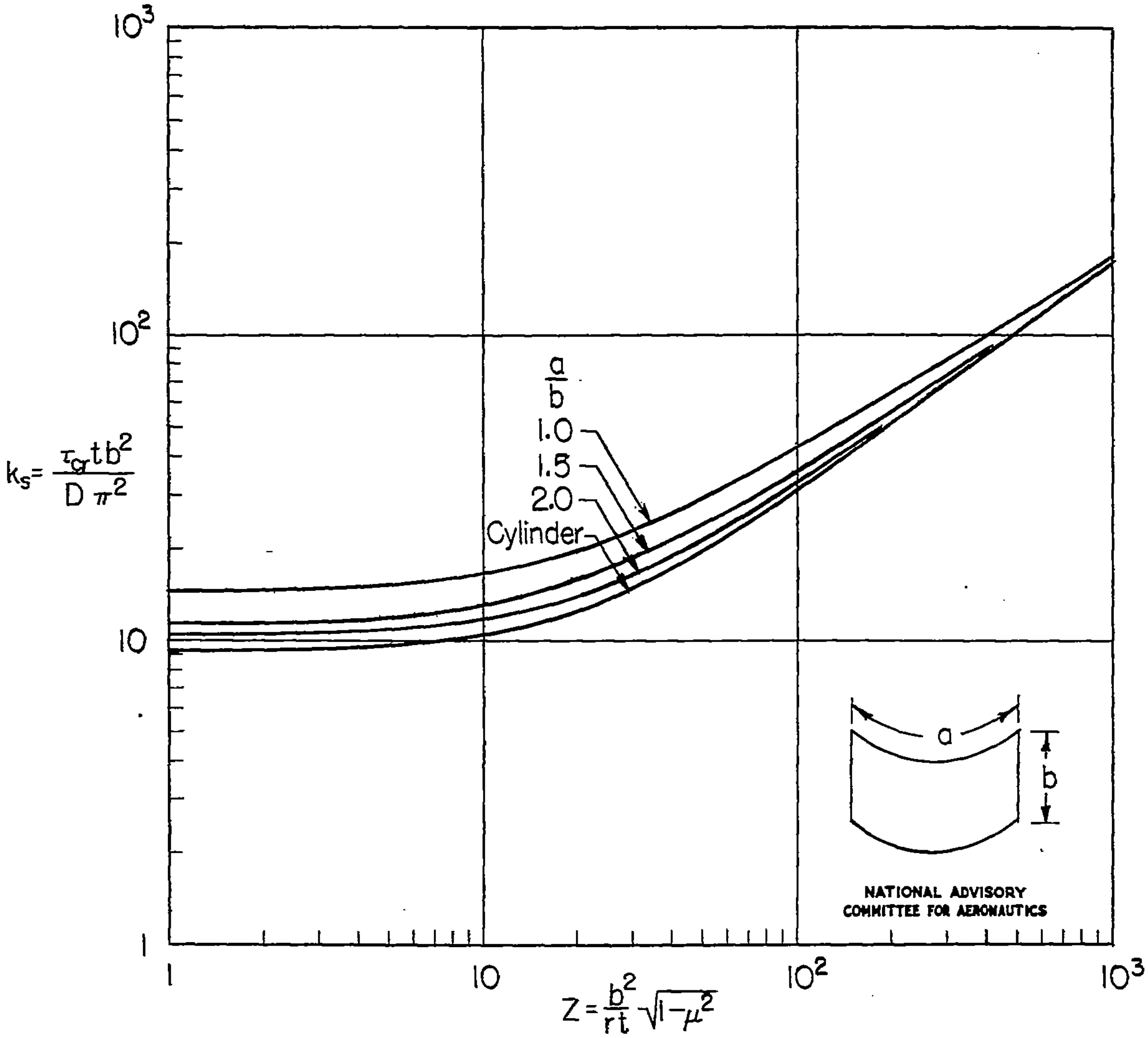


Figure 6.- Estimated theoretical critical-shear-stress coefficients for curved panels with clamped edges and having circumferential length greater than axial length.

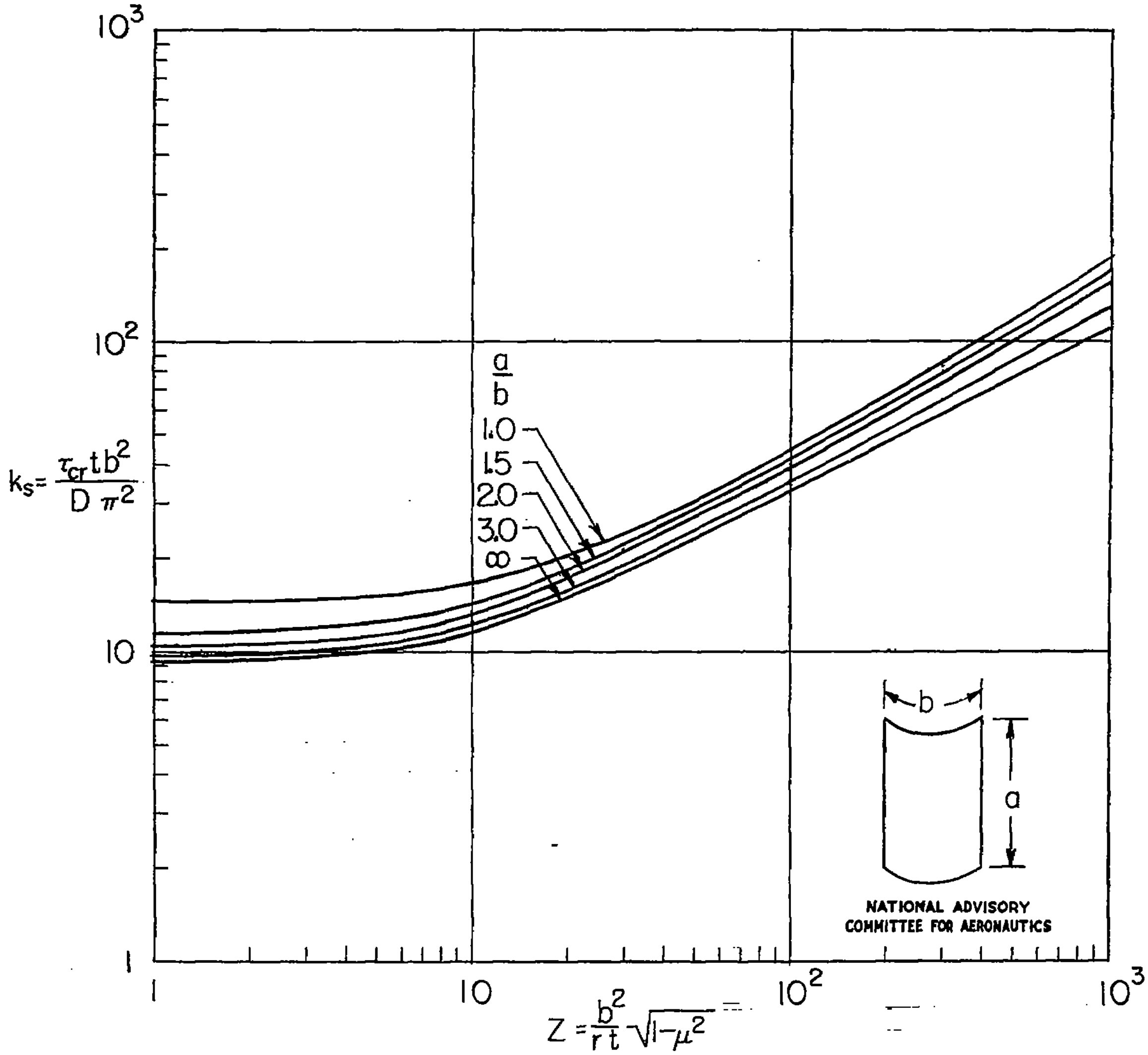


Figure 7.- Estimated theoretical critical-shear-stress coefficients for curved panels with clamped edges and having axial length greater than circumferential length.

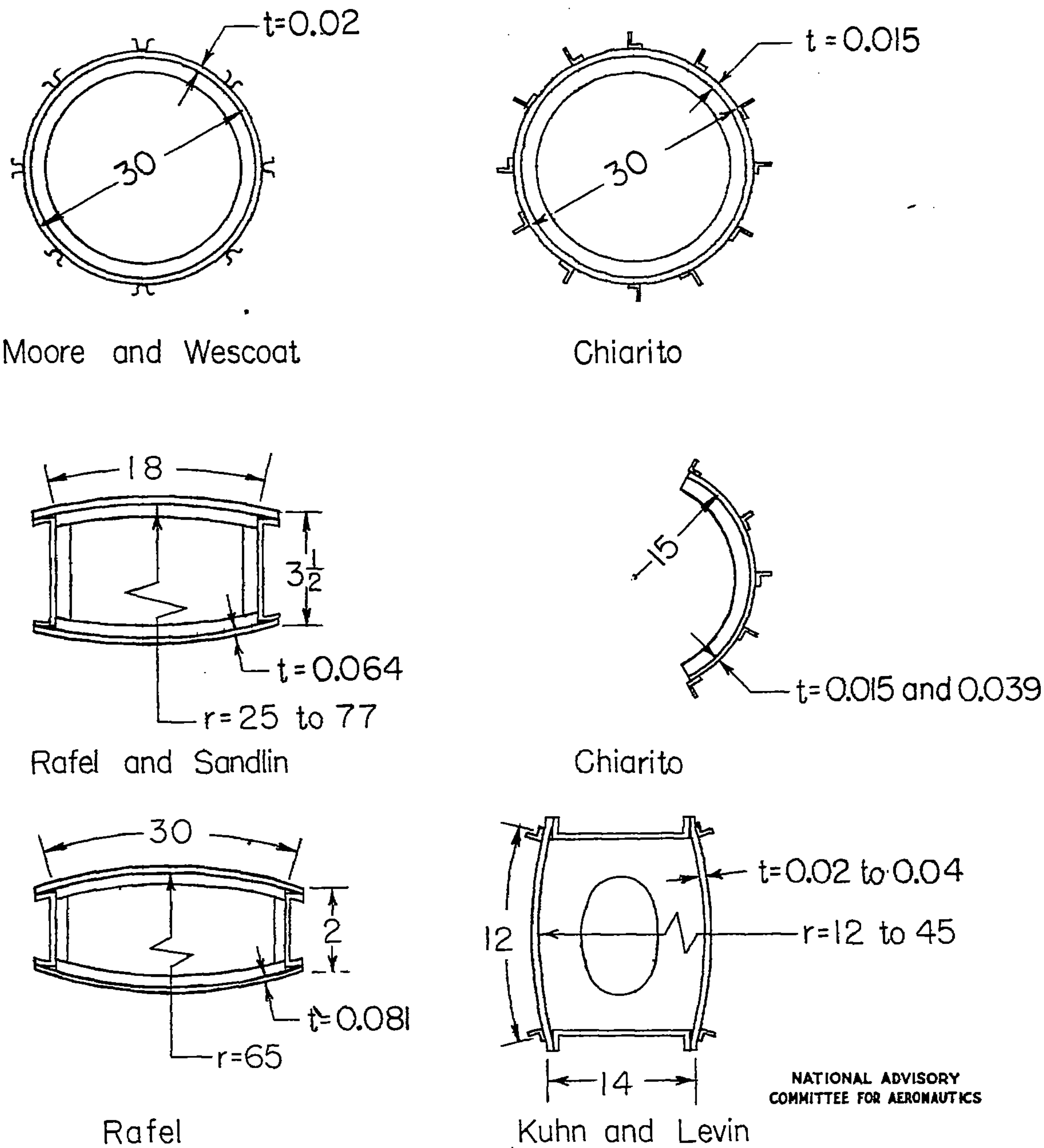
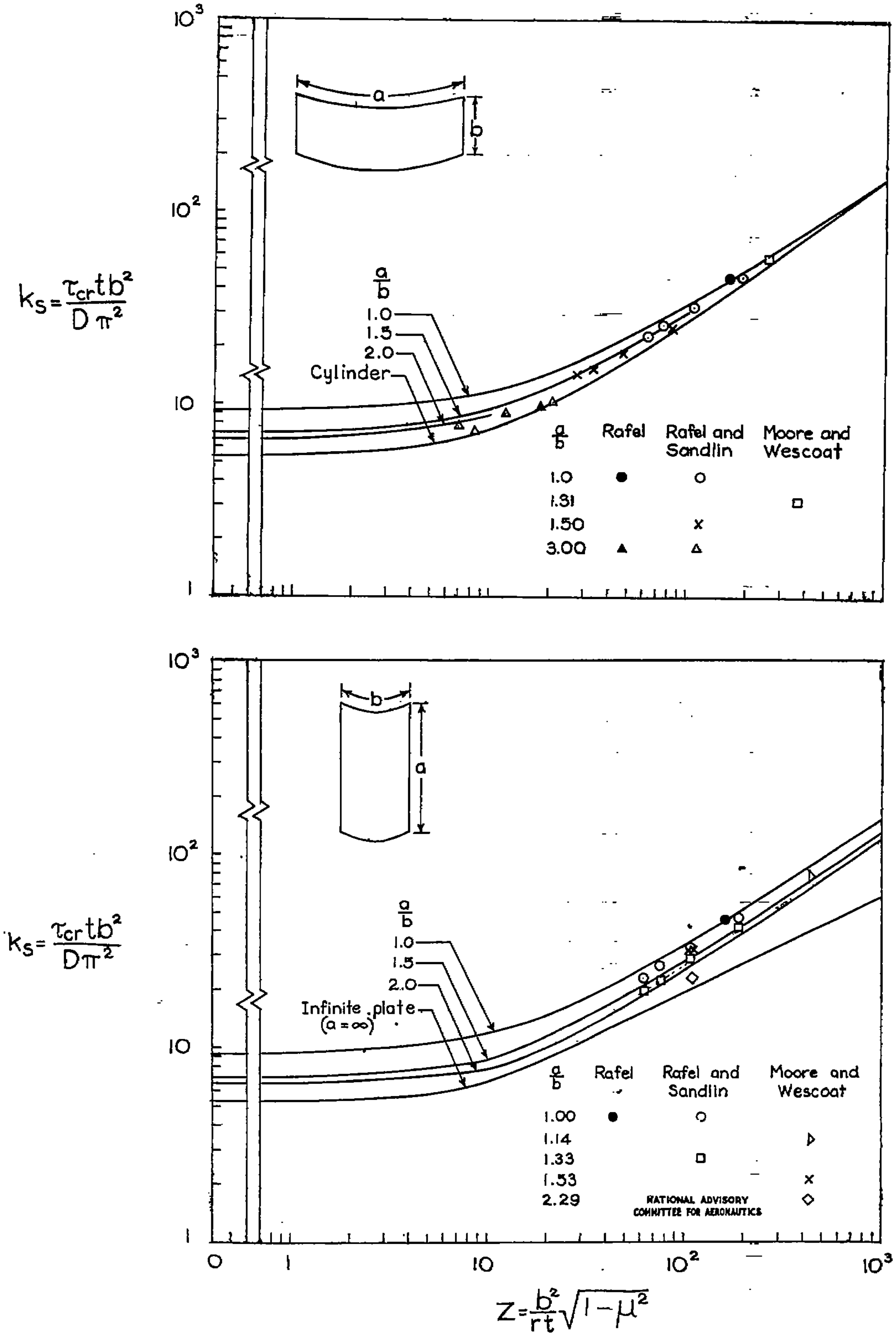
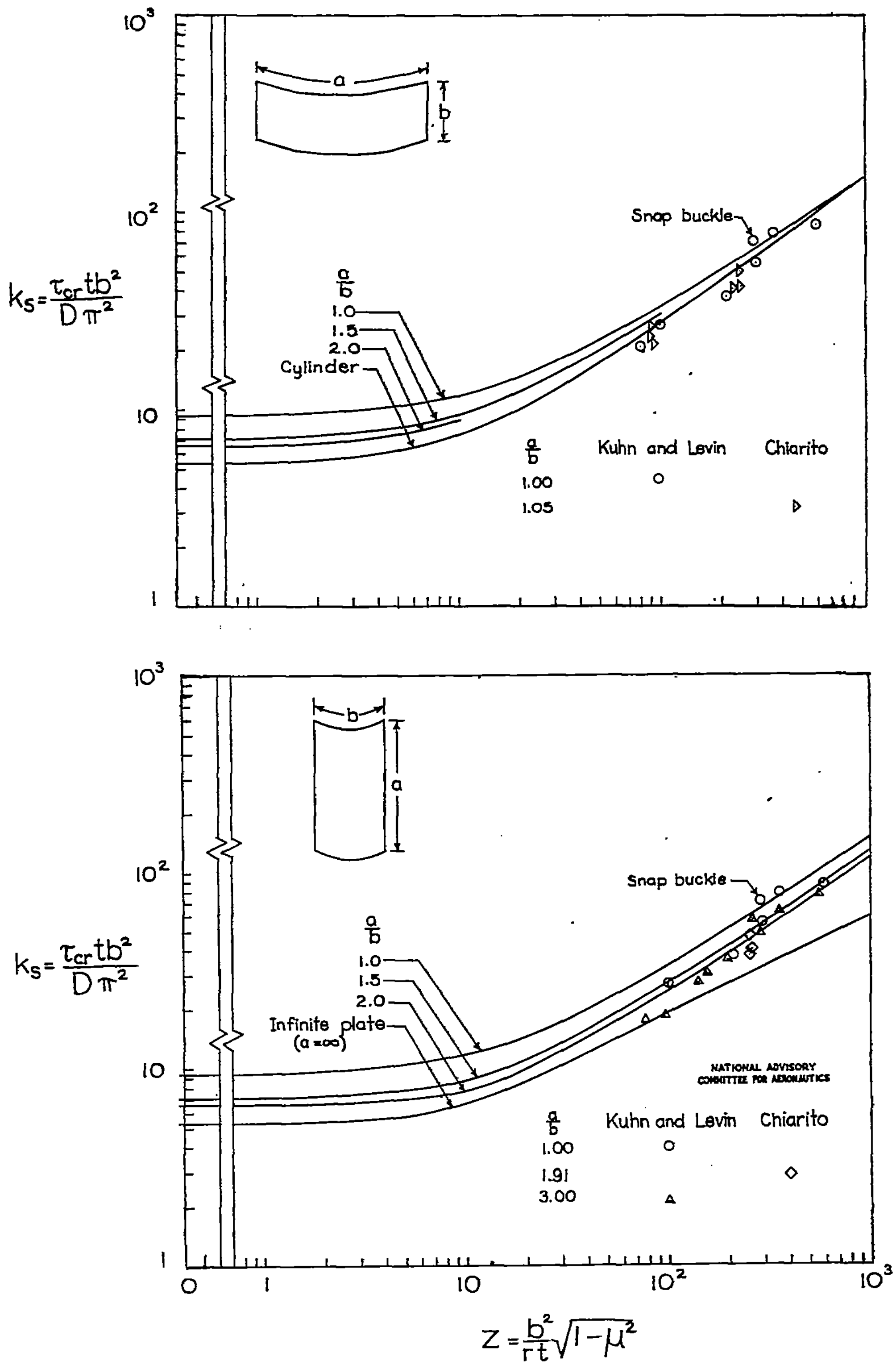


Figure 8.- Transverse sectional views of specimens used to determine critical shear stress of curved panels in various investigations. Different length-width ratios of panels were obtained by varying bulkhead or ring spacing and in some cases by spacing of longitudinal stiffeners.



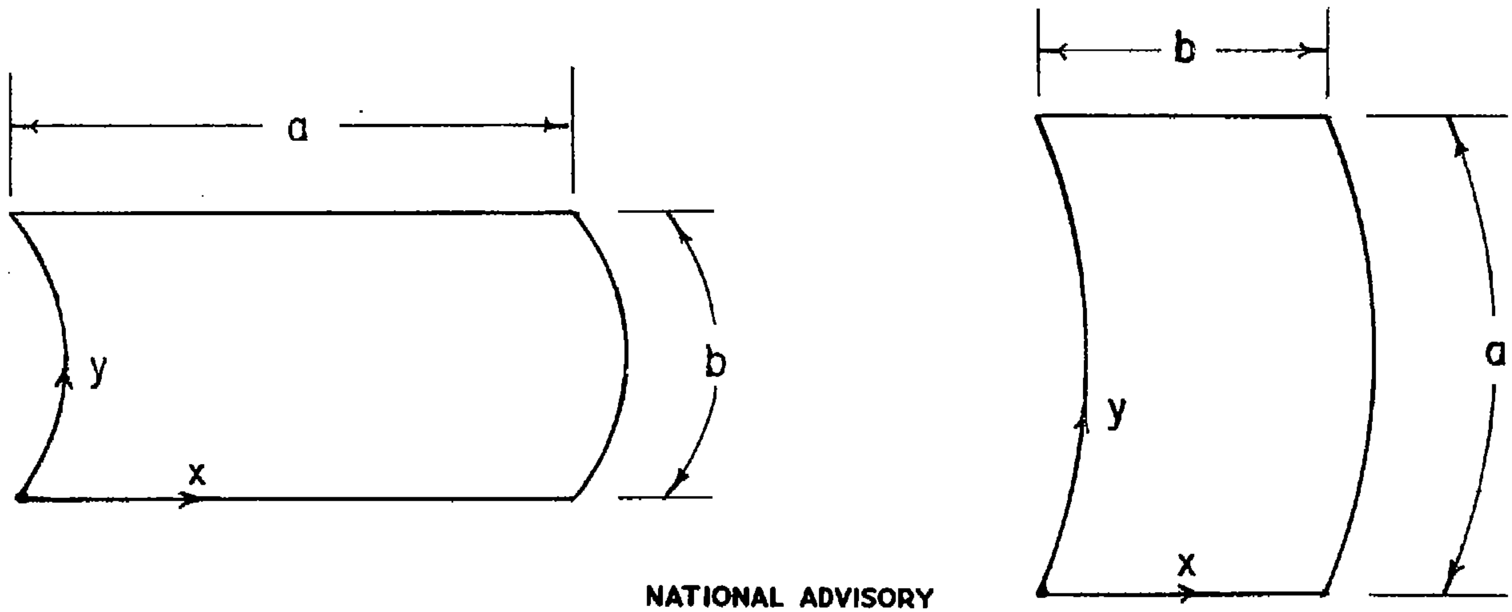
(a) Specimens slightly affected by initial eccentricities.

Figure 9.- Comparison of theoretical critical-shear-stress coefficients of simply supported curved panels with test results of other investigations.



(b) Specimens considerably affected by initial eccentricities.

Figure 9.- Concluded.



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(a) Panel long in axial direction.

(b) Panel long in circumferential direction.

Figure 10.- Coordinate systems used in theoretical solution.